

Combined constraints on the SUSY parameter space from Δr and Higgs boson search *

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Abstract

Combining the constraints coming from the M_W measurements and the unsuccessful search for the Higgs boson at LEP we determine in the framework of MSSM the allowed mass regions for the lighter scalar partner of the top quark. For a heavy top quark particularly strong bounds are obtained for low values of $\tan \beta \equiv v_2/v_1$ and light bottom squark.

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Recent calculation of the quantity Δr , i.e. of the W^\pm boson mass in terms of G_F , α_{EM} and M_Z – the three at present best measured electroweak observables, within the Minimal Supersymmetric Standard Model (MSSM) revealed [1, 2] that, for fixed m_t – the top quark mass, it is equal to or lower than Δr in the Standard Model (SM) with light, $\mathcal{O}(M_Z)$, Higgs boson ϕ^0 . There are two reasons for such behaviour of Δr in MSSM. Firstly, all genuine SUSY particles (like squarks, sleptons, charginos) give negative contributions to Δr . Therefore in order to maximize it one has to keep those particles very heavy, say $\mathcal{O}(1\text{-}2\text{ TeV})$. Their contribution to Δr is then negligibly small as follows from the Appelquist – Carazzone decoupling theorem. Secondly, as is well known [3], the Higgs sectors of MSSM and SM are different. All the masses and couplings of five MSSM Higgs bosons, h^0 , H^0 , A^0 and H^\pm , can be (at the tree level) parametrized by two variables only: M_{A^0} – the CP -odd Higgs boson mass and $\tan\beta$ – the ratio of the two vacuum expectation values of the two Higgs doublets. For given $\tan\beta$, maximum of Δr is attained for $M_{A^0} \gg M_Z$. In this limit, however, A^0 , H^0 and H^\pm effectively decouple and the Higgs sector of MSSM is SM-like with one light Higgs boson h^0 . For large $\tan\beta$, which maximize Δr , $M_{h^0} \sim \mathcal{O}(M_Z)$ thus explaining why the upper limit for Δr in MSSM does not exceed the SM value of Δr for $M_{\phi^0} \sim M_Z$.

When the SUSY particle masses are in the 100 GeV range their negative contribution to Δr may be substantial. Two major sources of such negative contributions have been identified. Both are related to rather large violation of the so called custodial $SU_V(2)$ symmetry by sparticle masses (much in the same way as it is violated by the top – bottom mass splitting leading to decrease of Δr as m_t grows). One is the slepton sector with sneutrinos lighter than M_Z . In such a case Δr receives large (larger for larger $\tan\beta$) negative contribution due to $SU_V(2)$ violating sneutrino – left handed charged slepton mass splitting. The second source is the similar mass splitting between stop and sbottom masses. The second effect, which is bigger for smaller values of $\tan\beta$, is more interesting because it grows with m_t which enters the formulae for the top squark masses.

With the new, improved, data: $M_Z = 91.1895 \pm 0.0044$ GeV [4] and $M_W = 80.23 \pm 0.18$ GeV [5], the quantity Δr

$$\Delta r = 1 - \frac{\pi\alpha_{EM}}{\sqrt{2}G_F} \frac{1}{M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right)} \quad (1)$$

is constrained to $\Delta r = 0.044 \pm 0.010$ (notice the change as compared with [1] where $\Delta r = 0.044 \pm 0.015$ which mainly reflects the improvement in M_W measurement). This means that at 1σ level the top quark mass in MSSM is constrained to $m_t < 177$ GeV. Though the 1σ bound on Δr may prove too restrictive, it is clear that for a heavy top quark very little room is left for acceptable negative contributions from squarks and/or sleptons.

This becomes particularly intriguing in view of the apparent discrepancy between the value of the top mass 174 ± 17 GeV reported recently by the CDF collaboration [7] and the measurement of $R_b \equiv \Gamma_{Z \rightarrow \bar{b}b} / \Gamma_{Z \rightarrow \text{hadrons}}$ at LEP [4] which deviates from its Standard Model prediction [6] with $m_t = 174$ GeV by 2σ . Such discrepancy may well be due to the existence of a light SUSY partner of the top quark [8]. In this context any constraint on the top squark mass coming from quantities other than R_b are very important.

In this note we want to explore more carefully such constraints coming from Δr and the fact that no SUSY scalar Higgs boson with mass less than roughly 50 GeV has been found so far [9]. From the analysis of Δr in MSSM [1] it follows that restricting Δr to its experimental 1σ bounds would imply that either SUSY particles, in particular, squarks, are very heavy so that they are not relevant for the LEP1 physics or that the top quark mass is smaller than the central value given by CDF measurement. Therefore, in what follows, as a useful guideline we will consider two values of m_t , 174 and 160 GeV together with the more conservative, 2σ bound on Δr for the former and 1σ bound for the latter. Of course, the same analysis can be applied to any other values of m_t and/or more/less restrictive bounds on Δr .

The second constraint, $M_{h^0} > 50$ GeV, becomes relevant because in MSSM quarks and squarks have important impact on the masses of the scalar Higgs bosons [10, 11]. Dominant corrections are induced by the top and stop loops and are large in the case of $m_t \gg M_W$. The general top squarks mass squared matrix has the form:

$$\mathcal{M}_{\tilde{t}_1, \tilde{t}_2}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 & -m_t A_t \\ -m_t A_t & M_{\tilde{t}_R}^2 \end{pmatrix} \quad (2)$$

where

$$\begin{aligned} M_{\tilde{t}_L}^2 &= m_Q^2 + m_t^2 - \frac{1}{6} \cos 2\beta (M_Z^2 - 4M_W^2)v \\ M_{\tilde{t}_R}^2 &= m_T^2 + m_t^2 + \frac{2}{3} \cos 2\beta (M_Z^2 - M_W^2) \end{aligned} \quad (3)$$

Its two eigenvalues $M_{\tilde{t}_1}^2$ and $M_{\tilde{t}_2}^2$ are the physical \tilde{t}_1 (lighter) and \tilde{t}_2 (heavier) top squark masses. As follows from the simple estimate

$$\Delta M_h^2 \sim \frac{m_t^4}{M_W^2} \log \left(\frac{M_{\tilde{t}_1} M_{\tilde{t}_2}}{m_t^2} \right) + \dots \quad (4)$$

in contrast with the case of top squarks heavier than top quark considered previously [10, 11, 13], for top squarks lighter than the top quark the corrections to the Higgs boson masses can be negative. The complete formulae for the corrections to the Higgs boson masses in the, so called, effective potential approximation can be found in e.g. [11]. For our purpose this method is

accurate enough as follows from the detailed comparison of its results with the full diagrammatic 1-loop calculations [12] the error of the approximation being smaller than 3–5 GeV.

For low values of $\tan\beta$, for which the tree level lighter Higgs boson mass $M_{h^0}^{tree}$ is small (it is bounded from above by $M_{h^0}^{tree} < |\cos 2\beta| M_Z$ and therefore is smaller than 50 GeV for $\tan\beta < 1.85$) the positive radiative corrections are crucial for making such small values of $\tan\beta$ still phenomenologically acceptable. Therefore, for given m_t the requirement $M_{h^0} > 50$ GeV puts some restrictions on the top squark masses.

The ellipses in eq.(4) stand for other terms which, in particular, tend to decrease $\Delta M_{h^0}^2$ in presence of large *left-right* mixing in the top squarks sector i.e. when $m_t A_t \sim M_{\tilde{t}_{L,R}}^2$. This is very important in view of the fact that large *left-right* mixing decreases the absolute value of the negative contribution of light squarks to Δr and therefore seems to be necessary for making Δr acceptable.

If $A_t = 0$, i.e. in the case of no *left-right* mixing in the stop sector (mixing in the sbottom and other sectors can be neglected if naturality is invoked) mass eigenstates are simply \tilde{t}_L and \tilde{t}_R with $\tilde{t}_1 \equiv \tilde{t}_L$ or \tilde{t}_R depending on which one is lighter. $M_{\tilde{t}_L}$ is related to $M_{\tilde{b}_L}$ given by

$$M_{\tilde{b}_L}^2 = m_Q^2 + m_b^2 - \frac{1}{6} \cos 2\beta (M_Z^2 + 2M_W^2) \quad (5)$$

since they contain one common free soft SUSY breaking parameter m_Q^2 . Therefore the lower bound on the bottom squark mass of 120 GeV puts also lower bound on the mass of purely left handed stop which depends on the top mass and $\tan\beta$. In general, because $m_t > M_W$ we always have $M_{\tilde{t}_L} > M_{\tilde{b}_L}$ and this mass splitting is bigger for smaller values of $\tan\beta$.

To demonstrate the importance of the Higgs boson mass bound we plot in Fig.1 the lower limit for the mass of the lighter top squark $M_{\tilde{t}_1}$ as a function of the mass of the left-handed sbottom (we neglect the small *left-right* mixing in the bottom squark sector) for few values of $\tan\beta$ and two values of the top quark mass. For fixed $M_{\tilde{b}_L}$ a scan is performed over $M_{\tilde{t}_R}$ and A_t and the lowest value of $M_{\tilde{t}_1}$ compatible with the requirement $M_{h^0} > 50$ GeV is plotted. We see that particularly strong bounds are obtained for $1 < \tan\beta < 2$. For $\tan\beta > 2.5$ the constraint quickly disappears. This is because the tree level part of M_{h^0} grows with $\tan\beta$ (for $M_{A^0} \gg M_Z$ we have $M_{h^0} = |\cos 2\beta| M_Z$) and therefore there is more room for negative contribution to M_{h^0} from radiative corrections. The maxima seen in Fig.1 for $\tan\beta = 1.2$ and 1.5 arise roughly at the points where $M_{\tilde{t}_R}^{min}$, for which the minimum of $M_{\tilde{t}_1}$ compatible with the requirement $M_{h^0} > 50$ GeV is attained, becomes equal to $M_{\tilde{t}_L}$. To the left of the maximum $M_{\tilde{t}_R}^{min} > M_{\tilde{t}_L}$ and $M_{\tilde{t}_1}$ grows with $M_{\tilde{t}_L}$ (which is related to $M_{\tilde{b}_L}$). For larger values of $\tan\beta$ we always have $M_{\tilde{t}_R}^{min} < M_{\tilde{t}_L}$ and no maxima appear.

On the top of the constraints on the lighter stop mass following from the requirement $M_{h^0} > 50$ GeV come the constraints from M_W measurement. However, before we study the impact of the latter, we would like to improve the calculation of Δr done in ref. [1] including corrections to the Higgs boson masses. In refs. [1, 2] the effects of these correction on Δr have not been taken into account. However, although they are formally a two-loop effects, they should be included, especially for low values of $\tan\beta$ for which the corrections to the lighter Higgs boson mass can reach even 80 GeV [10] and exceed many times the tree level mass itself¹. Most easily they can be taken into account in the effective potential approach. For given values of the top and stop masses, using the formulae given in ref. [11] we calculate the corrected values of M_{h^0} and M_{H^0} as well as the corrected mixing angle α . These corrected quantities are subsequently used in formulae for the gauge boson self-energies as given e.g. in [13] instead of the tree level ones. Such a procedure is consistent as far as gauge boson self-energies are concerned and does not spoil the on-shell renormalization scheme.

The dependence of Δr on the MSSM Higgs sector parameters M_{A^0} and $\tan\beta$ with (solid lines) and without (dotted lines) radiative corrections to the scalar Higgs boson masses and couplings included is illustrated in Fig.2. In Fig. 2a $m_t = 175$ GeV and $M_{\tilde{t}_1} \sim M_{\tilde{t}_2} \sim 1000$ GeV so that the corrections to M_{H^0, h^0} are positive. In Fig. 2b $m_t = 160$ GeV and $M_{\tilde{t}_1} = 60$ and $M_{\tilde{t}_2} \sim 150$ GeV leading to negative mass corrections. In both cases the *left-right* mixing vanishes and masses of the charginos, neutralinos, sleptons and squarks from the first two generations as well as the mass of the right handed sbottom are taken of order $\mathcal{O}(1 \text{ TeV})$. As can be seen, the corrections are more important for the case with heavy top squarks and small values of the $\tan\beta$. In these plots we do not care about h^0 mass apart from requiring it to be greater than zero. This eliminates the $M_{A^0} < 185$ GeV part of the curve for $\tan\beta = 1.5$ in Fig. 2b. Fig. 2a confirms the estimates given in [1] that for large $\tan\beta$ (for which in MSSM the upper bound of Δr is reached) the corrections to Δr are of order 0.001. Notice, that the corrections do not change the fact that for fixed $\tan\beta$ maximum of Δr is reached in the limit of large values of M_{A^0} . Since in the same limit the mass of h^0 is maximized for fixed $\tan\beta$, in order to derive absolute bounds on the top squark masses, we will keep $M_{A^0} = 500$ GeV. For the same reason we will always keep charginos, neutralinos, sleptons as well as squarks from the two first generations of order $\mathcal{O}(1 \text{ TeV})$.

We turn now to the genuine stop/sbottom sector contribution to Δr . In the limit of no *left-right* mixing the influence of the \tilde{t}_R mass on Δr is only indirect, through corrections to the Higgs boson masses since purely

¹The inclusion of these corrections does not affect the overall limits on Δr (for fixed m_t) given in [1] because both, the lower and the upper bounds are attained for large $\tan\beta \gg 10$ for which the corrections have been estimate to be small – $\delta(\Delta r) \sim 0.001$.

right-handed sfermions decouple almost completely (apart from their very small contribution through $\Delta\alpha_{EM}$, see [1]) from Δr . In this limit Δr receives negative contribution from $\tilde{t}_L - \tilde{b}_L$ mass splitting which leads, for large m_t and light \tilde{b}_L , to unacceptably low values of Δr . More precisely, for example for $\tan\beta = 2$ and $M_{\tilde{b}_L} = 120$ (150) GeV ($M_{A^0} = 500$ GeV and other SUSY particles very heavy) Δr becomes smaller than 0.034 for $m_t = 154$ (158) GeV and smaller than 0.024 for $m_t = 173$ (178) GeV. For given m_t , Δr can be increased by taking \tilde{b}_L and \tilde{t}_L squarks heavy enough and/or switching on the *left-right* mixing which makes the negative contribution to Δr from stop/sbottom sector smaller in absolute magnitude [1]. The typical features of the squarks contribution to Δr are illustrated in Figs. 3a-d where for $\tan\beta = 2$ and $m_t = 174$ GeV we plot Δr as a function of the lighter stop mass for four different values of $M_{\tilde{b}_L}$. At the rightmost point of each curve the *left-right* mixing vanishes (i.e. $A_t = 0$ there) and the corresponding mass of $M_{\tilde{t}_R}$ is used to label different curves. The *left-right* mixing (i.e. the parameter A_t) increases along the curves from the right to the left. If there were no corrections to the Higgs boson masses each curve would extend up to $M_{\tilde{t}_1} = 45$ GeV – the current direct experimental LEP limit. However, for too low values of $M_{\tilde{t}_1}$ and large *left-right* mixing in the stop sector, the negative corrections make $M_{h^0} < 50$ GeV and the curves are cutted off at the value of A_t for which M_{h^0} reaches 50 GeV. The increase of Δr with increasing $M_{\tilde{t}_R}$ at the rightmost points of the curves is also due to inclusion of radiative corrections to the Higgs sector.

As can be seen in Fig.3, for low values of $M_{\tilde{b}_L}$ imposing the constraint $\Delta r > 0.024$ eliminates completely curves for small $M_{\tilde{t}_R}$ strenghtening therefore the lower bound on $M_{\tilde{t}_1}$. For given $M_{\tilde{b}_L}$ and $M_{\tilde{t}_R} > M_{\tilde{t}_L}$ the lighter stop mass is always bounded from above by $M_{\tilde{t}_L}$ itself because it is \tilde{t}_L which plays the role of \tilde{t}_1 in the limit of vanishing *left-right* mixing. For $M_{\tilde{b}_L} < 130$ GeV however, this upper bound on $M_{\tilde{t}_1}$ is shifted to the lower values by the requirement $\Delta r > 0.024$. The magnitude of this shift decreases with increasing $M_{\tilde{t}_R}$. Therefore, the overall upper bound on $M_{\tilde{t}_1}$ (for fixed $M_{\tilde{b}_L}$) depends on how big hierarchy $M_{\tilde{b}_L} \ll M_{\tilde{t}_R}$ is allowed by the naturality criterion. In order not to be too restrictive when producing the overall bounds on $M_{\tilde{t}_1}$ we took 1 TeV as an upper limit for $M_{\tilde{t}_R}$.

Scanning over values of $M_{\tilde{t}_R}$ and A_t for given $M_{\tilde{b}_L}$ one arrives at the absolute bounds on $M_{\tilde{t}_1}$. These bounds are demonstrated in Fig. 4a for $m_t = 174$ GeV (and 2σ constraints imposed on Δr) and in Fig. 4b for $m_t = 160$ GeV (with Δr restricted to 1σ) for few values of $\tan\beta$. As has been discussed above, for low values of $M_{\tilde{b}_L}$ (close to 120 GeV) the upper bound on $M_{\tilde{t}_1}$ comes from the lower (2 or 1σ) bound on Δr . This effect is not very big for $m_t = 174$ GeV – for $\tan\beta = 2(1.5)$ $M_{\tilde{t}_1}^{max}$ is lower than $M_{\tilde{t}_L}$ by 5 (15) GeV for $M_{\tilde{b}_L} = 120$ GeV and disappears for $M_{\tilde{b}_L} > 130$ (140) GeV – but is more pronounced for $m_t = 160$ GeV

where for $\tan\beta = 2$ (2.5) $M_{\tilde{t}_1}^{max}$ is lower than $M_{\tilde{t}_L}$ by 27 (20) GeV for $M_{\tilde{b}_L} = 120$ GeV and these effect extends up to $M_{\tilde{b}_L} = 170$ (155) GeV.

The lower limit for $\tan\beta = 1.5$ comes solely from the condition $M_{h^0} > 50$ GeV (and is therefore given by the same lines as in Fig.1). However, for $\tan\beta > 1.75$ due to the restriction imposed on Δr the lower bound on $M_{\tilde{t}_1}$ is stronger than it follows from the requirement $M_{h^0} > 50$ GeV only. It is also interesting to notice that for $m_t = 160$ GeV and $\tan\beta = 1.5$ (1.75) the combined constraints from M_{h^0} and Δr eliminate values of $M_{\tilde{b}_L}$ up to 155 (130) GeV.

To conclude, taking into account constraint coming from M_W measurement by UA2, CDF and D0 and from the unsuccessful MSSM Higgs boson search at LEP1, we have obtained rather strong (for small values of $\tan\beta$) constraints on the allowed regions in the $(M_{\tilde{b}_L}, M_{\tilde{t}_1})$ plane. Let us stress, that since we have kept all other SUSY particles very heavy, the obtained allowed regions are maximal. Making e.g. sneutrinos light would result in much stronger constraints on the $(M_{\tilde{b}_L}, M_{\tilde{t}_1})$ plane because of the additional negative contributions to Δr . On the other hand it is hard to imagine that sleptons are much heavier than squarks (actually, all the renormalization group study of the MSSM embeded into supergravity predict the opposite). Therefore, our bounds are certainly the weakest ones.

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FIGURE CAPTIONS

Figure 1.

Lower bounds on the lighter top squark mass from $M_{h^0} > 50$ GeV for different values of the $\tan\beta$ (marked on the curves) as a function of $M_{\tilde{b}_L}$ - the mass of the left handed sbottom and two different masses of the top quark.

Figure 2.

Dependence of Δr on M_{A^0} for different values of $\tan\beta$. Solid (dotted) lines show the results with corrections to the Higgs boson masses included (neglected).

a) $m_t = 174$ GeV and $M_{\tilde{t}_1} \sim M_{\tilde{t}_2} \sim 1000$ GeV.

b) $m_t = 160$ GeV and $M_{\tilde{t}_1} = 60$ and $M_{\tilde{t}_2} \sim 150$ GeV.

All other SUSY particles are heavy, $\mathcal{O}(1 \text{ TeV})$.

Figure 3.

Δr as a function of the lighter stop mass, $M_{\tilde{t}_1}$ for $m_t = 174$ GeV, $\tan\beta = 2$ and four different masses of the left handed sbottom $M_{\tilde{b}_L}$. Numbers on the curves are the corresponding values of $M_{\tilde{t}_R}$. The *left-right* mixing, i.e. A_t , vanishes at the rightmost point of each curve and increases from the right to the left. The curves are cutted off at the values of A_t for which M_{h^0} becomes lighter than 50 GeV. The 2σ lower limit on Δr is marked by dotted lines.

Figure 4.

Allowed regions in the $(M_{\tilde{b}_L}, M_{\tilde{t}_1})$ plane for different values of $\tan\beta$. For $m_t = 174$ GeV in Fig. 4a 2σ bound on Δr is imposed. For $m_t = 160$ GeV in Fig. 4b Δr is restricted to its 1σ experimental value. Allowed regions for $\tan\beta = 1.5, 1.75, 2$ and 2.5 are marked by dashed, dot-dashed, solid and dotted lines respectively.

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